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ON THE CONCEPT OF STABILITY OF INELASTIC SYSTEMS

by

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STABILITY OF INELASTIC SYSTEMS
By D. C. Drucker¹ and E. T. Onat²

Abstract

Simple models are employed to bring out the large and important differences between buckling in the plastic range and classical elastic instability. Static and kinetic criteria are compared and their inter-relation discussed. Non-linear behavior in particular is often found to be the key to the physically valid solution. The non-conservative nature of plastic deformation in itself or in combination with the non-linearity requires concepts not found in classical approaches. Conversely, the classical linearized condition of neutral equilibrium is really not relevant in inelastic buckling. Plastic buckling loads are not uniquely defined but cover a range of values and are often more properly thought of as maximum loads for some reasonable initial imperfection in geometry or dynamic disturbance.

The models indicate that basically the same information is obtained from essentially static systems by assuming initial imperfection in geometric form as by assuming dynamic disturbances. One approach complements the other and both are helpful in obtaining an understanding of the physical phenomena.

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Introduction

In the analysis of structures, as in most branches of engineering and science, problems are solved by simplifying them enormously. The geometry of the structure, the material of which it is composed, and the loads applied are all strongly idealized. A rather extreme example is a riveted truss which is often analyzed as pin-connected and as though all axial forces in the members were applied along the assumed straight centerline of each and were uniformly distributed over the cross-section. Justification of any idealization requires an analysis of the real system and an interpretation of the results. Should reasonable deviations produce large changes in the result the idealization is not permissible. Clearly, in the example just cited the truss members will have appreciable end moments so that the stress will not be distributed uniformly. Large stress concentrations will occur at rivet holes and other discontinuities. In a strict sense the idealizations are entirely in error. From the practical point of view, however, if allowable stresses are based on experience as codified in engineering specifications, it will often be found that the simplified analysis is adequate.

The basic elastic buckling problem of the Euler strut is more relevant to the present discussion. The assumptions made are that the column is perfectly straight, that the load is static and is applied along the centerline, and that the material is homogeneous, linearly elastic, and free of initial stress. Computation of critical load is made with a linearized instead of exact expression for curvature while a condition of neutral equilibrium is sought under constant

load. Neutral signifies that equilibrium is possible in neighboring deformed positions as well as in the ideal configuration. An equivalent approach is to compute the static force or to compute the impulse needed to displace the strut laterally and then to define the critical load as the load at which zero disturbance is needed.

The terms instability and buckling load can thus be given precise meaning for an ideal Euler elastic strut without any consideration of the behavior of real columns. However, an engineer of skeptical turn of mind encountering such a calculation for the first time could well be excused if he ignored the results completely. Indeed they were generally ignored until aircraft sections emphasized their significance. There is no obvious or intuitive reason for accepting the validity of the idealizations. Rolled and extruded sections have appreciable initial stress and are imperfect in shape and form. Loads are not absolutely axial nor are they ordinarily constant in magnitude.

Fortunately, in this simple problem, idealizations and imperfections can be studied analytically. When Euler solved the problem originally, he used the exact expression for curvature so that its linearization is known to be permissible. Also, the eccentrically loaded, imperfect column can be treated. The usefulness of the Euler critical load computation lies in the fact that the calculations based on real columns show the critical load to be a reasonable limiting load as long as the stresses induced are below the elastic limit and the imperfections are small, Fig. 1.

Similar types of calculations have proved equally successful for other structural elements where small deviations from the idealized condition also do not produce large differences in behavior. In a way, this successful analysis of linearized, idealized, and simplified systems was unfortunate. It seems to have led to the idea that the classical linearized theory with its static criterion of neutral equilibrium solved the real problem. As an actual imperfect system is so troublesome, even in the elastic range, it is easy to see how such a situation could develop. When low values were obtained experimentally for the buckling loads of cylindrical and spherical shells the warning was not taken seriously in general. Donnell did attribute the discrepancy to initial imperfection but Kármán and Tsien* explained the result with a large deflection theory, Fig. 2. Although the two approaches are comparable in some ways, it was the buckling computation which had the more popular appeal.

The picture began to change when Shanley** introduced the concept of considering the loading process itself by returning to the strut problem and following what happens as the load is increased and the elastic limit is exceeded. He demonstrated conclusively that the classical buckling approach, which gives the critical load P_K is not appropriate for the plastic range. Perhaps, however, too much attention was paid to the remarkable proof that an initially perfect column could start to bend at the tangent modulus load, P_T . The

*Th. V. Kármán and H. Tsien, Jour.Aero.Sci. Vol 7, pp 43-50, 1939

**F.R. Shanley, Jour.Aero.Sci. Vol 13, p. 678, 1946
Jour.Aero.Sci. Vol 14, p. 261, 1947

buckling aspects of Shanley's problem seemed to overshadow the maximum load computation, or more generally, his load-deflection relation, possibly because the spread between P_T and P_K is so small for a column. As already mentioned, initial imperfections are always present so that basically the question of practical importance is how the deflection grows with load and what is the maximum load, P_M , which can be carried. Pearson* contributed to the overall picture by proving that Shanley's load-deflection curve for an initially straight column is the limiting load-deflection curve for an imperfect column as the deviation from straightness approaches zero, Fig. 3.

Onat and Drucker** added the more elaborate example of a cruciform column in the plastic range which fails by twisting, Fig. 4. This plate problem solved by small deflection theory demonstrated the need to take into account what might be termed small but finite deformations which occur as the loading proceeds if extremely small initial imperfections are present. Here the spread between P_M and P_K is large and of real practical significance similar in a way to the shell problem of Fig. 2.

Ziegler*** focussed attention on dynamic loading or dynamic disturbance superposed on static loading and pointed up the shortcomings

*C.E. Pearson, Jour.Aero.Sci. Vol 17, p. 417, 1950

**E.T. Onat and D.C. Drucker, Jour.Aero.Sci. Vol 20, pp.181-186, 1953

***H. Ziegler, Ingenieur-Archiv, Vol 20, pp.49-56, 1952

H. Ziegler, Zeitschrift für angewandte Mathematik and Physik, Vol 4, Fasc. 2 and 3, 1953

of classical theory. A kinetic criterion of instability can be employed to obtain correct answers for systems with perfect geometry. The necessity or convenience of a kinetic criterion for essentially static problems, however, remains to be considered.

To summarize these introductory remarks, the real problem of instability involves:

- 1) imperfection of geometry of structure
- 2) imperfection of loading
- 3) dynamic disturbances
- 4) inhomogeneity, residual stresses, and similar ever-present imperfections

In general calculations must be made taking into account the effect of small but finite deformations and possibly large deformation as well in a few cases. On the other hand, classical linearized theory for the idealized system computes instability for infinitesimal deformation or dynamic disturbance under constant load. If the loading is conservative and the system is linearly elastic, such an analysis is often satisfactory. The same critical load is obtained for both infinitesimal deformation and dynamic disturbance. As inelastic systems are both non-linear and non-conservative (path dependent), it is not reasonable to expect the classical linear type of theory to produce significant results.

In the Shanley example Fig. 3, it is non-linearity which is responsible for deviation from straightness at the tangent modulus load, non-conservatism which produces P_M . Non-linearity is so prominent in the elastic-plastic range because a relatively small amount of deformation produces the change from elastic response with its high

modulus of elasticity to the plastic response at low modulus.

The meaning of instability is itself often obscure and a matter of new definition. Loads between P_T and P_K , Fig. 3 are stable in one sense unstable in another. Dynamic disturbances and geometric imperfections must be considered and the history of the deformation must be followed.

Non-Linearity, Static Analysis

As stated plastic action is always both non-linear and non-conservative. The first model to be considered, Fig. 5, has both characteristics but it is non-linearity which provides the interesting features of its behavior. Rod OA is rigid, the pin at O is frictionless and the spring attached at A has an elastic and a plastic range of force F shown schematically by the full line in Fig. 6. Small displacements only will be analyzed so that $L\theta$ replaces $L \sin \theta$, $L \cos \theta$ is taken as L for the lever arm of F , and the position of A below its maximum possible height is approximated by $L\theta^2/2$. Upon unloading, the force-displacement curve is straight and parallel to the original elastic line as shown. The actual force displacement curve will be replaced by the broken line segments BC, CD, DD', etc. for convenience of description and of algebraic manipulation.

The mathematical expression of the simplified force-displacement relation depends upon whether the material is behaving elastically or plastically and is

$$dF = k_t L d\theta \quad (1)$$

or

$$dF = kLd\theta \quad (2)$$

Expression (1) applies for $FdF > 0$ and $\theta > \theta_1$ on first loading or more generally $FdF > 0$ and $|F| > F_y$ and greater than any value reached previously. Expression (2) applied for $FdF < 0$ or for $|F| < F_y$ or any previously attained value.

Suppose that the spring is so adjusted that $F = 0$ when $\theta = 0$. The bar OA will then be in equilibrium in the vertical position for all values of load, P . The question of the stability of the equilibrium is not as trivial as might at first appear. If the usual static criterion is employed, OA is rotated through a very small angle θ and the work done by P , $PL\theta^2/2$ is compared with the energy stored by the spring, $FL\theta/2$ where $F = kL\theta$. Equating the two terms leads to the critical load

$$P_K = kL \quad (3)$$

The Shanley concept of increasing P as θ is introduced does not change the result at all. It is the stability of deflected positions which is the key to the physically significant behavior of the system. Equilibrium requires $PL\theta = FL$ or

$$F = P\theta \quad (4)$$

For the elastic range, $\theta < \theta_1$, $F = kL\theta$ and $P = kL$. When θ reaches or exceeds θ_1 , $F = F_y + k_t L(\theta - \theta_1)$ where $F_y = kL\theta_1$.

$$\begin{aligned}
 P\theta &= kL\theta_1 + k_t L(\theta - \theta_1) = (k - k_t)L\theta_1 + k_t L\theta \\
 &= kL\theta - (k - k_t)L(\theta - \theta_1)
 \end{aligned} \tag{5}$$

as depicted in Fig. 7 by lines $\theta_0 = 0$.

If the spring is so adjusted that $F = 0$ at $\theta = \theta_0$,

$$F = kL(\theta - \theta_0) = P\theta$$

and

$$\theta = \frac{\theta_0}{1 - \frac{P}{kL}} = \frac{\theta_0}{1 - \frac{P}{K}} \tag{6}$$

in the elastic range. Beyond F_y , where as shown dashed in Fig. 7a

$$\frac{P}{P_K} = \frac{\theta_1}{\theta_0 + \theta_1}$$

$$F = kL\theta_1 + k_t L(\theta - \theta_0 - \theta_1) = P\theta$$

so that to maintain equilibrium P would have to decrease as θ increases. The maximum value of P therefore is given by

$$\frac{P_M}{P_K} = \frac{\theta_1}{\theta_0 + \theta_1} \tag{7}$$

with the restriction $P_M \geq k_t L$.

In a sense Figs. 7(a) and 7(b) which differ only in horizontal scale are a graphical representation of the two extreme possibilities. If θ_1 is large compared with probable initial and subsequent deflections $P = kL$ is a valid buckling load and will be found experimentally. On the contrary, if θ_1 is very small compared with probable initial deflection or equivalent eccentricity or

inclination of load P, the experimentally determined critical load or the one found in practice will be close to $k_t L$. When $k_t \ll k$, the difference between the extremes is very large. To this extent, the physical phenomenon may be said to be strongly dependent upon initial imperfection. In particular cases as for the cruciform, Fig. 4, it may turn out that the maximum load is bounded quite closely in practice.

If the model is taken as important in itself instead of simply a schematic illustration it is interesting to see how the extreme cases arise. $F_y = kL\theta_1$ may be rewritten as

$$\theta_1 = \frac{F_y}{kL} = \frac{\delta_e}{L} \quad (8)$$

where δ_e is the maximum elastic elongation of the spring. A long soft spring will provide a δ_e of several inches and P_k will be obtained. Short snubbing bars or wires, on the other hand, permit a δ_e of the order of thousandths of an inch and unavoidable measures in the bar and the loading will give an effective θ_0 many times θ_1 . P_M will then tend to be close to $k_t L$.

Although Fig. 6 shows the unloading behavior to be non-conservative, nothing in this section involved unloading so that a non-linear elastic material would exhibit exactly the same effects.

Non-Linearity, Kinetic Analysis

The model of Fig. 5 will now be considered from the alternative kinetic point of view. Now the question to be asked is whether a dynamic disturbance applied to a perfectly aligned system in

equilibrium produces bounded or unbounded oscillations within the framework of small deflection theory. The previous static analysis makes it quite clear that the answer will again depend upon the magnitude of the disturbance just as the size of θ_0 was significant.

Suppose that under constant load P , with $\theta = 0$, the bar OA is given an initial velocity $\dot{\theta}_0$. Conservation of energy gives, for small θ ,

$$I(\dot{\theta}^2 - \dot{\theta}_0^2) = PL\theta^2/2 - kL\theta^2/2 \quad (9)$$

for $\theta \leq \theta_1$

and

$$I(\dot{\theta}^2 - \dot{\theta}_0^2) = PL\theta^2/2 - [kL\theta_1^2/2 + kL\theta_1(\theta - \theta_1) + k_tL(\theta - \theta_1)^2/2] \quad (10)$$

for $\theta > \theta_1$ and $\dot{\theta} \geq 0$. I is the moment of inertia of the bar OA about O.

The system is unstable, that is motion in the initial direction will not stop, $\dot{\theta} > 0$, if the energy stored and dissipated in the spring can not be as large as

$$I\dot{\theta}_0^2 + PL\theta^2/2 \quad (11)$$

Obviously, from (9) and (10) $P < P_K = kL$ is a requirement for stability. At $P = P_K$, the slightest $\dot{\theta}_0$ will cause collapse, at $P > P_K$ the system runs away quickly. A plot of $\dot{\theta}$ vs θ is most convenient for exhibiting such properties, Fig. 8.

If $P < k_tL$, there is no $\dot{\theta}_0^2$ which can not be absorbed in accordance with small deflection Equation (10). The system is

therefore stable for $P < k_t L$ and the $\dot{\theta}$, θ diagram is composed of portions of ellipses, Fig. 9. For $k_t L < P < kL$ there will be levels of initial disturbance above which the system will not recover. Below these values of $\dot{\theta}_0$ the bar will come to rest and return. It is clear on physical grounds that on the return motion the system will again stop and oscillate back and forth. Unless the system collapses on the first try it is stable, Fig. 10. It is for this reason that on this and analogous cases when the external forces applied are conservative, static and kinetic analyses are equivalent.

Fig. 10 also shows the effect of combinations of initial velocity and initial displacement.

Non-Conservative Aspects

Fig. 11 shows the model employed by Duberg and Wilder*, following closely the model and concept of Shanley, to investigate the effect of initial imperfection in columns. DLGR is rigid and G is constrained to move vertically only by means of a frictionless guide so that the system has two degrees of freedom y , θ . L and R rest on two identical short bars of work-hardening material whose force-displacement diagrams are each idealized as two straight lines, Fig. 12. If a bar has been loaded to B and loading continues

$$F = F_B + k_t (\delta - \delta_B) \quad (12)$$

while if load is removed

*J.E. Duberg and T.W. Wilder, Jour. Aro. Sci. Vol 17, P.323, 1950.

$$F = F_B - k (\delta_B - \delta) \quad (13)$$

The static analysis is well known and basically has already been described in Fig. 3 where δ should be replaced by θ . The load rotation relation for the simplified model is given in Fig. 13. As initially imperfect system, $\theta = \theta_0$, will rotate more as load is applied and will have deviated appreciably from the vertical by the time the tangent modulus load, P_T , Fig. 14, is reached.

$$P_T = \frac{k_t b^2}{L} \quad (14)$$

A perfect system can deviate from the upright position at P_T and the slope of the P, θ line will be progressively flatter the larger the value of P at which rotation is permitted (see line segments between P_T and P_K of Fig. 3). At the reduced modulus load $P = P_K$, Fig. 14, the slope would be zero

$$P_K = \frac{k_t b^2}{L} \frac{2k}{k_t + k} \quad (15)$$

The third critical load of interest is the Euler elastic buckling load which can be obtained by putting $k_t = k$ in (14) or (15).

$$P_E = \frac{k b^2}{L} \quad (16)$$

The preceding discussion has implicitly assumed that P_T and therefore P_K and P_E are above the yield force $2F_y$, Fig. 12. Should P_T be less than $2F_y$, the initial imperfections may be small enough so that the material may not be made aware of the plastic deformation it could experience. Nothing unusual will then happen at the tangent

modulus load. The obvious example is a perfectly plastic or non-work-hardening material, $k_t = 0$, for which $P_T = 0$. In what follows, therefore, $P_T > 2F_y$.

A dynamic investigation of the geometrically perfect model is more interesting than the well-explored static study. As in the analysis of the first model, the consideration of a kinetic criterion of stability leads to investigation of the motion which follows arbitrary initial disturbances, and the question to be answered is whether or not the ensuing motion is limited. The system has two degrees of freedom, one of motion of G vertically, y , and one of rotation, θ , and the arbitrary initial dynamic disturbance will be characterized by $\dot{y}_0, \dot{\theta}_0$ where dot indicates time derivative. This pair of values determines the initial instantaneous center of rotation C and of course the initial angular velocity of the rigid bar $DLGR$. If the x -coordinate of C is between L and R , one supporting bar will start to unload and the other to load. On the other hand, if the x -coordinate of C lies outside L to R both bars will start to load or unload depending upon the sign of $\dot{\theta}_0$.

Designating the kinetic energy of the system by T , for small displacements the change in kinetic energy is

$$\Delta T = T - T_0 = P(y + \frac{L\theta^2}{4}) + \int F_L V_L dt + \int F_R V_R dt \quad (17)$$

where F and V are the instantaneous values of force and velocity at L and R . Denoting the bar constants by k_L and k_R , each of which may be either k or k_t , and considering the very beginning of the motion, first order terms cancel out and

$$\Delta T = \frac{PL\theta^2}{4} - \frac{k_L}{2} \left(y + \frac{b}{2}\theta\right)^2 - \frac{k_R}{2} \left(y - \frac{b}{2}\theta\right)^2. \quad (18)$$

The model therefore behaves like a linear elastic system at the beginning of the motion except that the non-conservative stress-strain relations determine the proper value of the k 's. A decrease in kinetic energy, $\Delta T < 0$, for all possible choices of y and θ indicates that the system is probably stable. An increase does not necessarily indicate true instability because even within the limitation of small deflection theory non-conservatism means that (18) need not hold as the motion proceeds.

If both support bars load, $k_L = k_R = k_t$ and $y \geq \frac{b}{2}\theta$. Therefore from (18), ΔT will be negative (indicating stability) for $P < P_T = \frac{k_t b^2}{L}$. For $P > P_T$ there will be a set of initial disturbances for which the kinetic energy of the system will increase at the beginning of the motion.

If $\dot{\theta}_0$ is positive and one support bar unloads while the other increases its load, $k_L = k_t$, $k_R = k$, and $y < \frac{b}{2}\theta$. With these conditions Equation (18) shows that ΔT will be negative for $P < P_K = \frac{k_t b^2}{L} \frac{2k}{k_t + k}$. The kinetic energy of a disturbance of this type will not grow until the load exceeds the reduced modulus load. What is of possibly greater significance here is that for $P > P_K$, $\Delta T > 0$ and the model will collapse because the k 's do not change as the motion proceeds.

If the artificial case is chosen in which both bars unload initially, $k_L = k_R = k$ and $y < -\frac{b\theta}{2}$. ΔT will be negative for $P < P_E = \frac{kb^2}{L}$. Such slowing down does not necessarily indicate

stability in this case. Obviously, after reversing its motion, the system will collapse if $P > P_K$.

In general, therefore the complete motion of the system must be examined under all possible disturbances to understand the significance of the critical loads obtained. As illustrated in Fig.15 because k_t is constant the motion settles down when $P_T \leq P \leq P_K$ whether ΔT is initially positive or negative. With the usual curved stress-strain diagram it is clear that collapse would occur for P less than P_K by an amount depending upon the magnitude of the initial values $\dot{\theta}_0$, \dot{y}_0 . Nevertheless it is interesting and important to note that all three critical loads, P_T , P_K , and P_E can be obtained from an analysis of a geometrically perfect system subjected to infinitesimal dynamic disturbance.

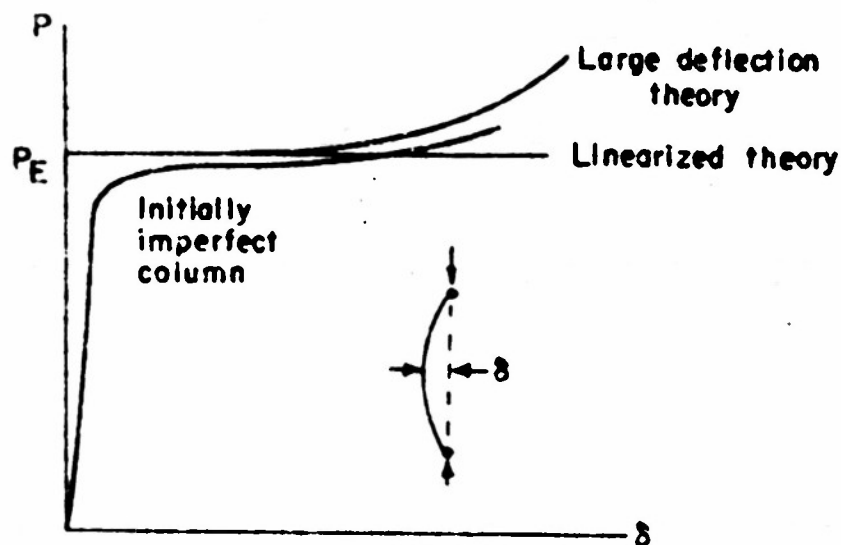


FIG. 1
ELASTIC STRUT

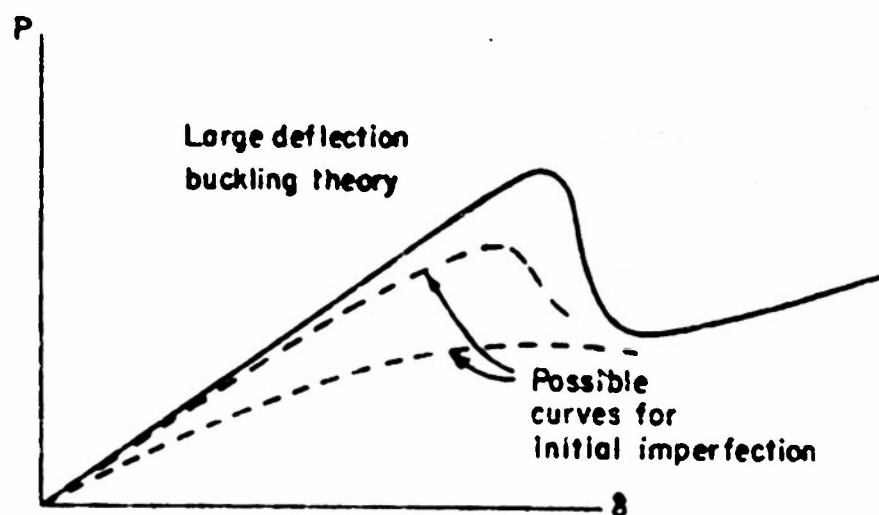
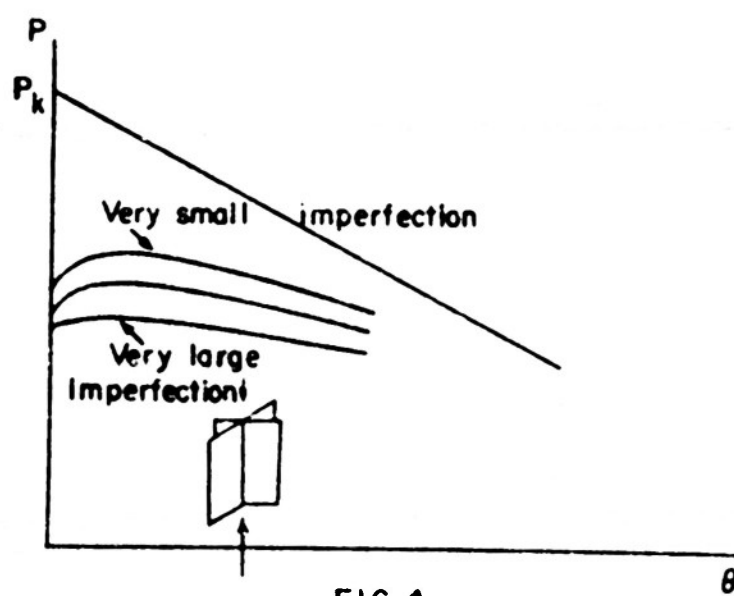
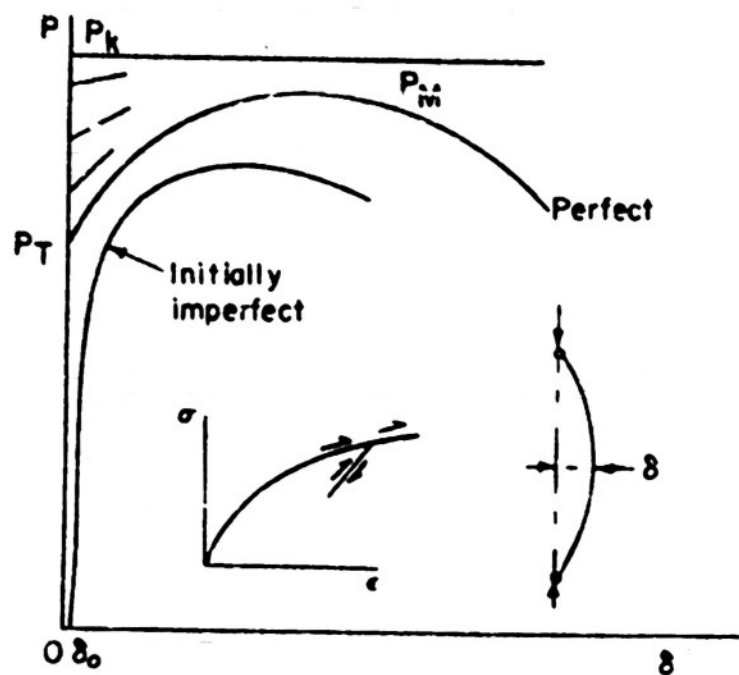


FIG. 2
ELASTIC SPHERICAL SHELL UNDER EXTERNAL
HYDROSTATIC PRESSURE OR CYLINDER UNDER END COMPRESSION



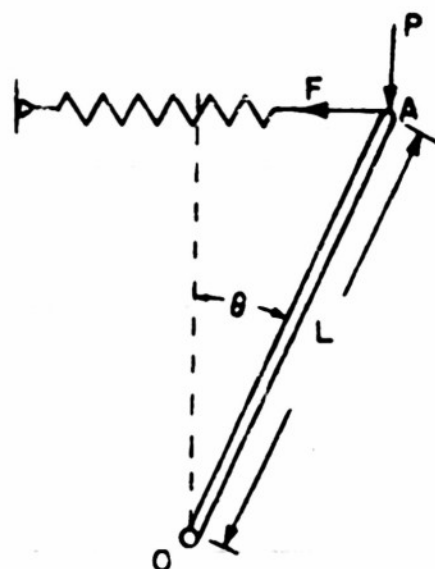


FIG. 5

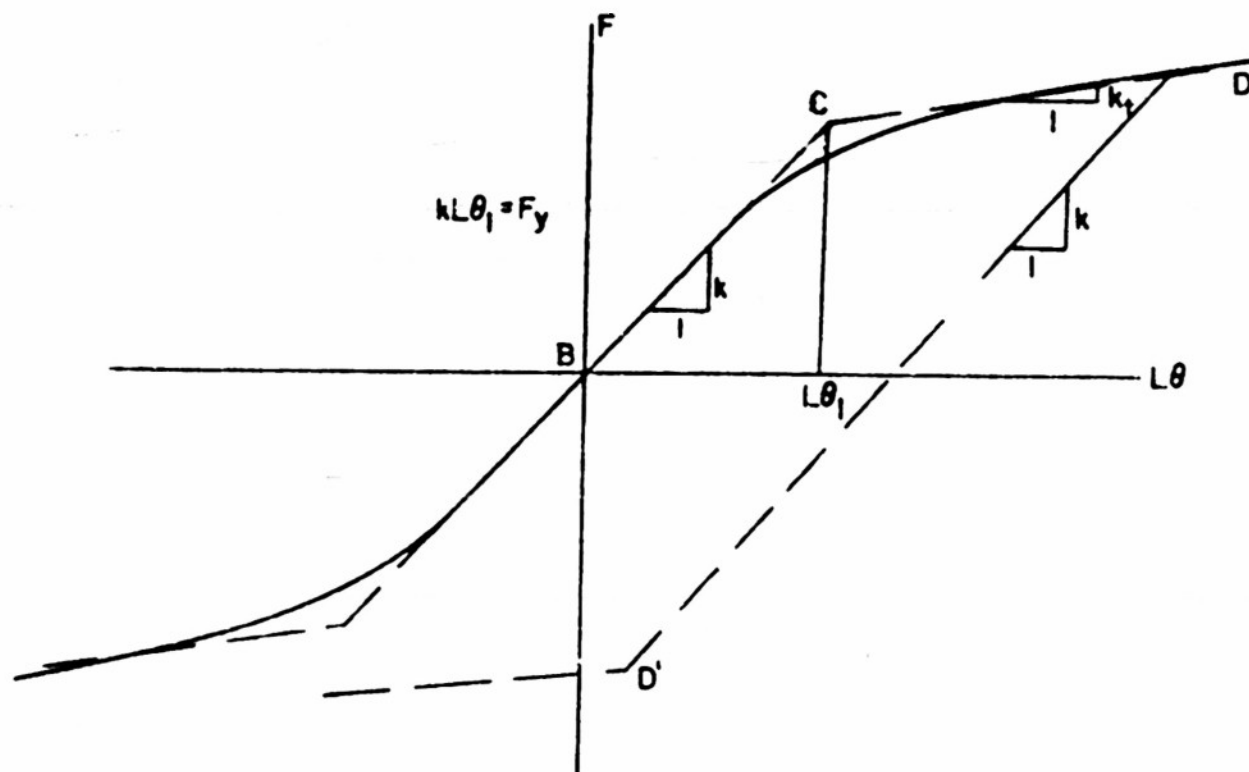


FIG. 6

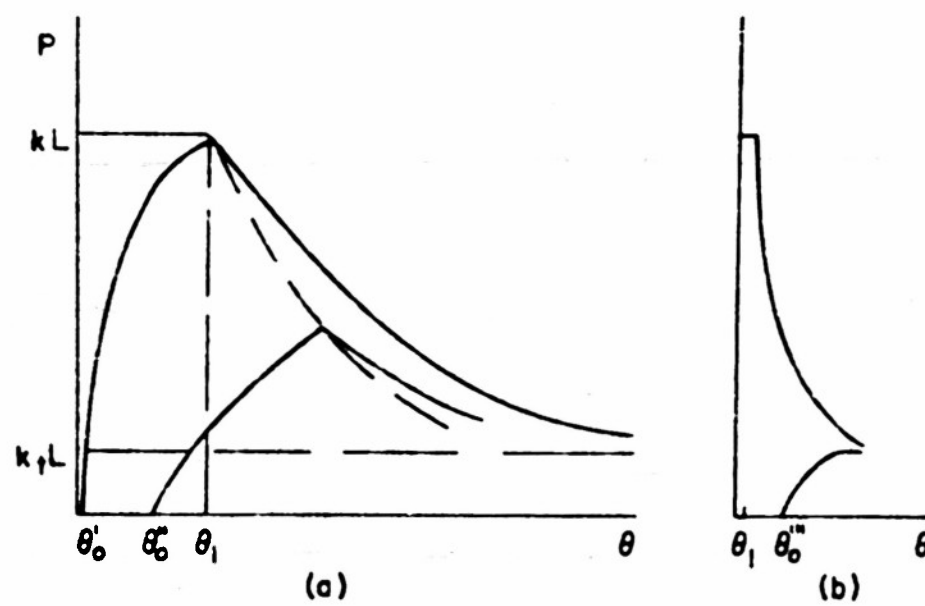


FIG. 7

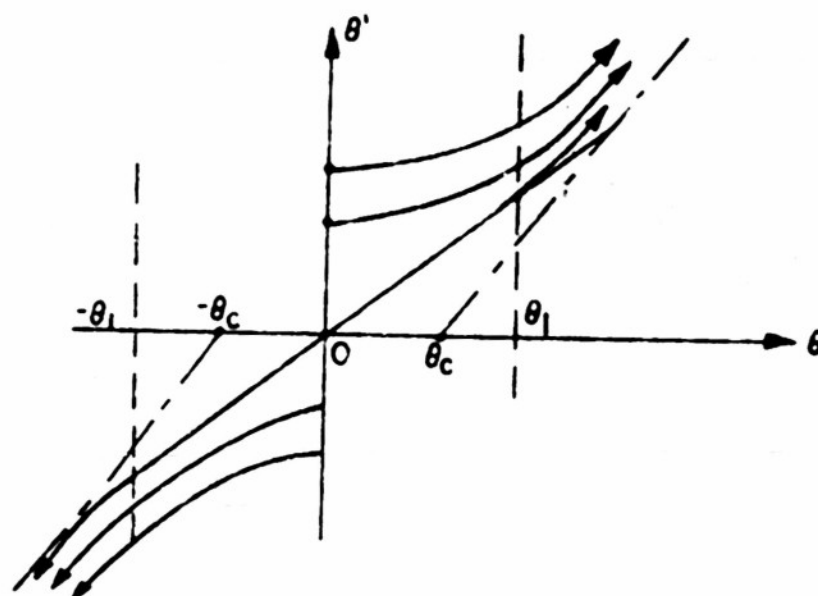


FIG. 8

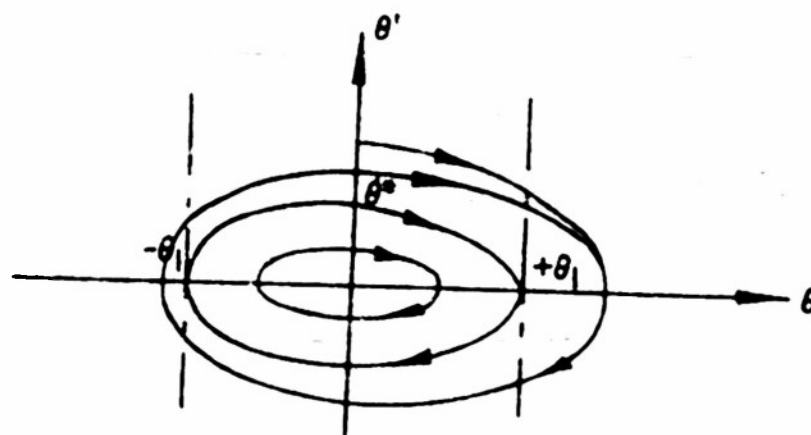


FIG. 9

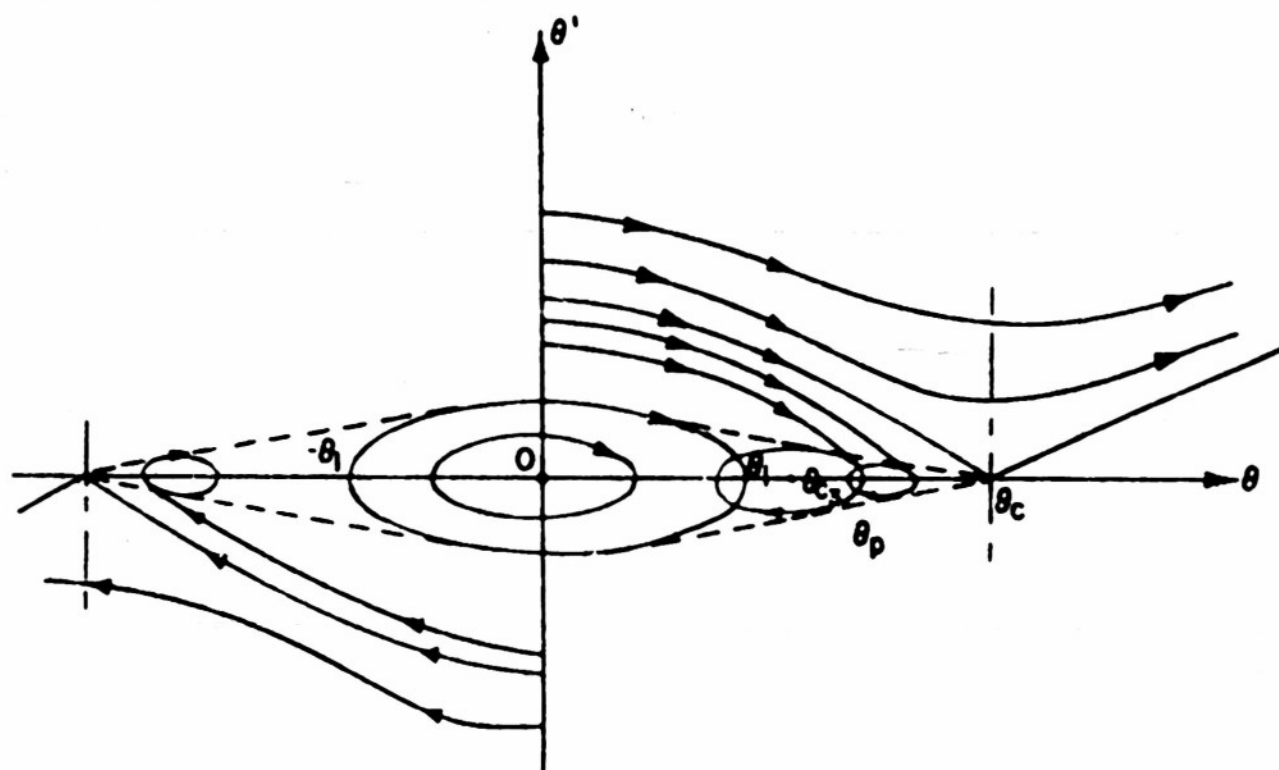


FIG. 10

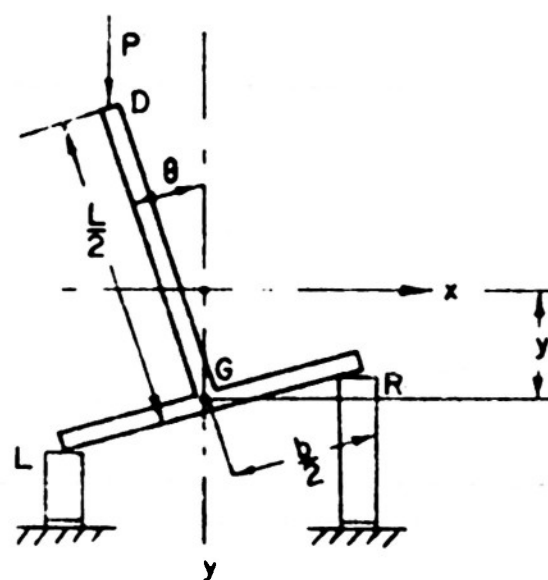


FIG. II

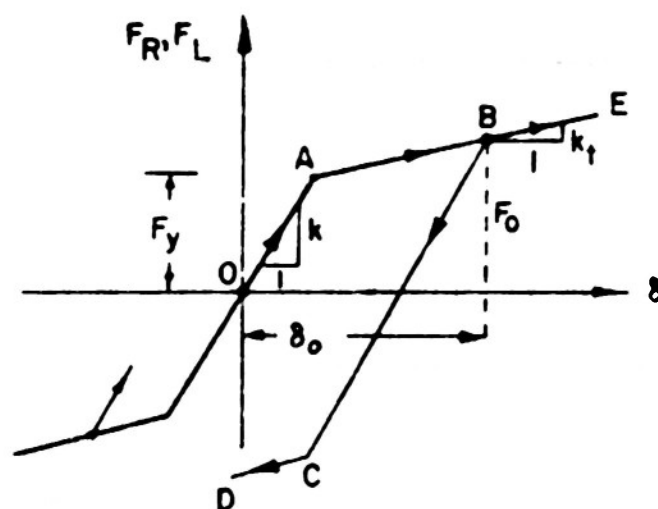


FIG. 12

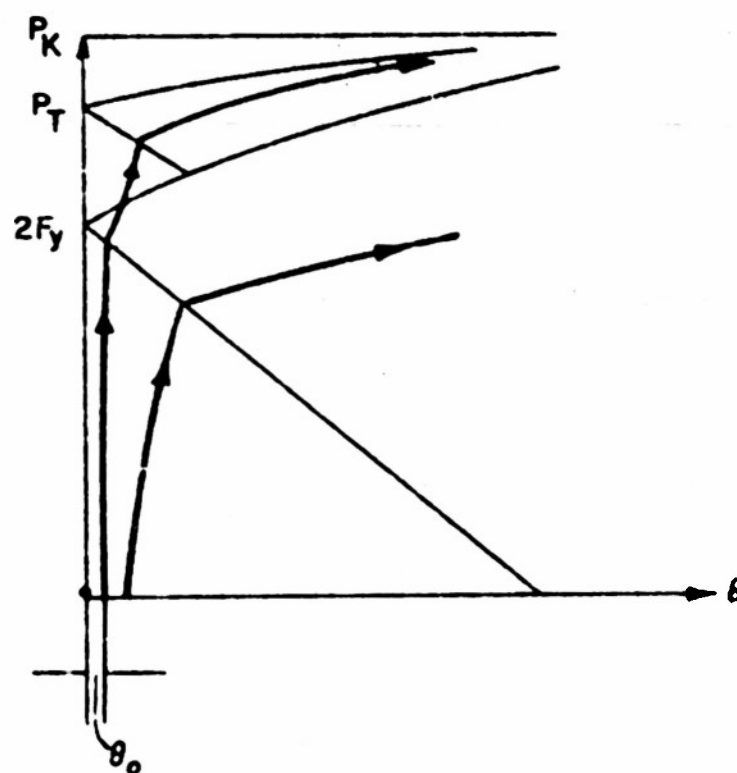


FIG. 13

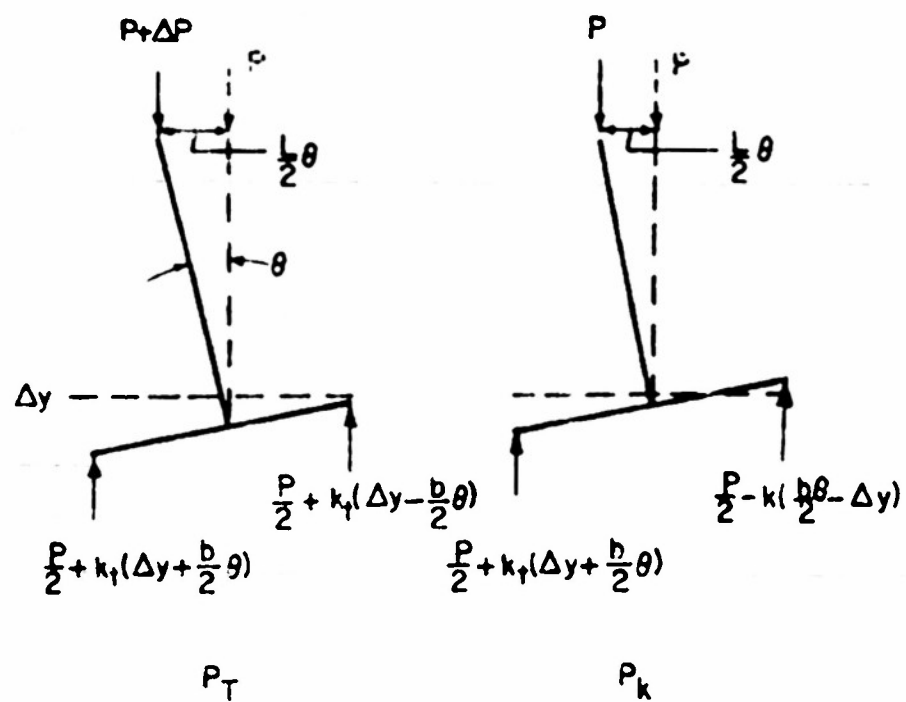


FIG. 14

TANGENT AND REDUCED MODULUS LOAD
COMPUTATION

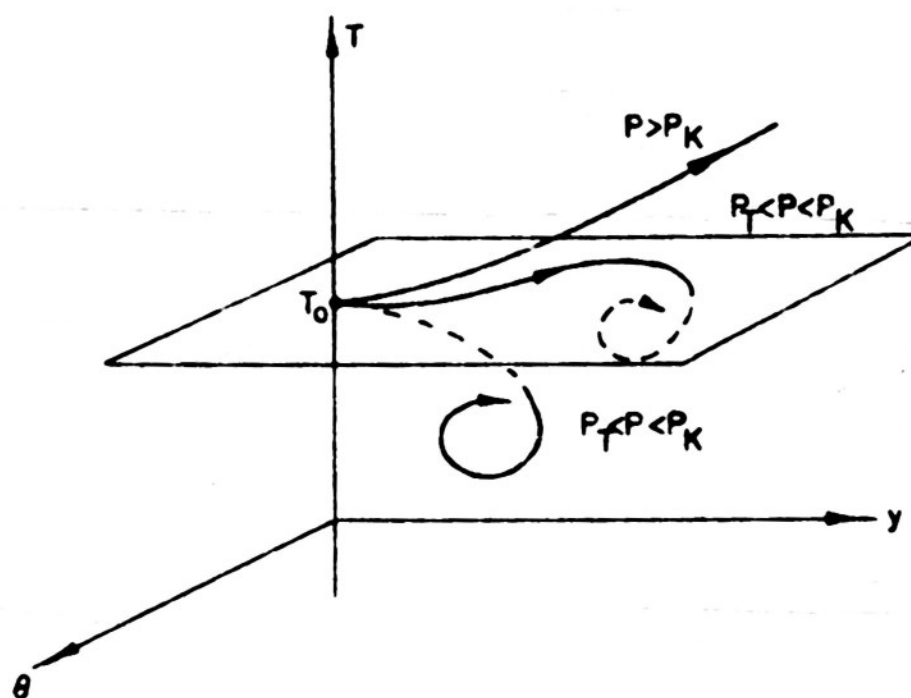


FIG. 15

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